**ICS 271**
**Fall 2017**
**Instructor : Kalev Kask**
**Homework Assignment 5**
**Due Thursday November 16**

1. (15) Consider a vocabulary with only four propositions, $A$, $B$, $C$ and $D$. How many models (satisfying true/false assignments) are there for the following sentences:

   (a) $A \vee B$

   (b) $(A \wedge B) \vee (B \wedge C)$

   (c) $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$

2. (15) Consider the statement "The car is either at Fred's house or at John's house. If the car is not at John's then it must be at Fred's house."

   (a) Describe a set of propositional letters which can be used to represent these statements.

   (b) Describe the statements using a propositional formula on the propositions you described for (a).

   (c) Can you determine where is the car?

3. (10) How would you use the truth table to prove that *modus ponens* is sound.

4. (10) Convert the following propositional calculus wff into CNF form:

$$\neg[((P \vee \neg Q) \rightarrow R) \rightarrow (P \wedge R)]$$

5. (20) Show how the $N$-Queens problem can be represented as a PSAT (propositional satisfiability) problem. (Hint: Introduce a proposition $q_{k,l}$ for each square $(k, l)$ of the $N \times N$ board. If $q_{k,l}$ has value $True$, there is a queen on square $(k, l)$; if it has value $False$, that square is empty. Now state the constraints of the problem in terms of these popositional symbols.)

6. (20) Use truth tables to show that the following sentences are valid, and thus that the equivalences hold. Some of these equivalence rules have standard names, which are given in the right column.

| | | | |
|---|---|---|---|
| $P \wedge (Q \wedge R)$ | $\Leftrightarrow$ | $(P \wedge Q) \wedge R$ | Associativity of conjunction |
| $P \wedge (Q \vee R)$ | $\Leftrightarrow$ | $(P \wedge Q) \vee (P \wedge R)$ | Distributivity of conjunction |
| $\neg(P \wedge Q)$ | $\Leftrightarrow$ | $\neg P \vee \neg Q$ | de Morgan's Law |
| $P \Leftrightarrow Q$ | $\Leftrightarrow$ | $(P \wedge Q) \vee (\neg P \wedge \neg Q)$ | |

7. (30) Look at the following sentences and decide for each if it is valid, unsatisfiable, or neither. Verify your decisions using truth tables, or by using the equivalences.

   (a) $Smoke \Rightarrow Smoke$

   (b) $Smoke \Rightarrow Fire$

   (c) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$

   (d) $Smoke \vee Fire \vee \neg Fire$

   (e) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \wedge Heat) \Rightarrow Fire)$

   (f) $(Big \wedge Dumb) \vee \neg Dumb$

8. (30) Trace the behavior of DPLL on the knowledge-base in Figure 7.16 (Russell and Norvig textbook) when trying to prove $Q$, and compare this behavior with that of forward chaining algorithm.